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Constructing a Heterogeneous Model for Soft Tissue Deformation Using Two Dimensional Wave Equations*

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***Abstract*— Soft tissue models play a vital role in the success of an inanimate training system, like the Virtual Reality training systems, which are gaining popularity for their potential for use in the medical industry. The accuracy of soft tissue models in such training systems allows for easy transference of skills to an operating room. This study aims to improve a soft tissue model, using two-dimensional wave equation, designed and discussed in a previous study. The model is used to mimic the interactions between soft tissue and a laparoscopic tool whereas in this study the aim is to develop a heterogeneous representation using the mathematical model. In this study, the soft tissue is modeled as a composite material; therefore, the articulation between the different layers is based on the mechanics of this material. A face validation study is performed to observe the accuracy of the new model with respect to another computer based model, finite element model. The results demonstrate the effectiveness of the heterogeneous representation of the soft tissue and its mechanics during manipulation.**

I. INTRODUCTION

Recent technological developments make Virtual Reality (VR) a very promising system for applications in the medical industry. The immersive technologies associated with VR have further enhanced the experience of the user therefore increasing its applications [1]. The growing interest in VR systems is a result of the success of flight simulators that are presently used to training pilots [2, 3]. The application of virtual reality to this industry was first suggested by Minsky in 1980 [1]. Some areas of applications, of this technology, are surgery, simulation, training, surgical training, telemedicine, rehabilitation and therapy [4-6]. The growing use of this technology in the medical industry is a result of the improving medical imaging technologies. This is because such advancements have led to more accurate collection of data that are associated with human anatomy of physiology for close to real representation in VR system [5].

A VR system is constructed using a number of components that determine the success of the system and its representation of a real world scenario. This study focuses on the soft tissue models and their behavior during real-time manipulations with an emphasis on the visualization and deformations. Real-time manipulations are of great interest in systems like a VR training system, a product the final soft tissue model is designed to be used in. The soft tissue of interest, in this study, is skin, but the method is applicable to any soft tissue using similar mechanical and material properties.

A. Mechanics of Skin

Skin plays an important role in human anatomy and physiology. The mechanical properties of this soft tissue are a result of the multiple layers of this soft tissue and their composition. Some of the functions are: protection of the visceral organs, insulation, and cushioning. The layers of skin that are of interest in this study are epidermis, dermis and subcutaneous fat. The epidermis is the top layer, dermis is the second layer and subcutaneous fat is the last layer. Each layer works with the others to achieve the functions of skin. Dermis plays an important role in the mechanical behavior of skin. This layer of the skin is made up of collagen and elastin fibers [7, 8].

Viscoelastic, heterogeneous and anisotropic are some of the major mechanical properties of skin [7]. Collagen and elastin fibers define the viscoelastic behavior of the skin. These fibers' composition and orientation determine the mechanical properties of the soft tissue [9]. Mechanical tests are performed in vivo on human skin using suction cups [9, 10]. These studies demonstrate the phases of stress-strain. The linear, initial elastic deformation is opposed by collagen fibers in the second stage and the last stage is the failure region [8, 9]. The connective tissue, muscles, and joints hold the skin in place resulting in a natural tension, in the soft tissue, which results in biaxial loading [11].

B. Soft Tissue Models in VR

There are two common soft tissue models used in VR for medical applications, which are spring-mass models and finite element analysis (FEA) models [12]. There are four factors are: stability of force output, smooth deformations of the tissue, consideration of necessary boundary conditions and demonstrations of accurate physical behavior in real-time [12, 13]. Mass-spring model is the most commonly used representation for real-time applications. Despite low accuracy, this model is preferable due to its considerably low computational cost. This model observes the viscoelastic properties of the soft tissue using the Kelvin-Voight model [14, 15]. The model is made up of nodes, representing the mass, and links, representing the spring, both of which are component that make up the mesh of the soft tissue [12]. On the other hand, the FEA models are very accurate but are limited by their computational cost. In this model, a mesh of soft tissue is deformed using differential equations of motion,

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which calculate the vector fields [12, 14]. The model related factors, accuracy and computational cost, constrain the soft tissue models in VR because they are directly proportional. This is seen as with accuracy the computational cost increases [12, 13]. These factors constitute the model of choice in real-time applications, which is the spring-mass model. This model observes the viscoelasticity of the tissue while disregarding its heterogeneity [16].

This study focuses on the manipulation of soft tissue models in VR due to the role of this behavior on the realism of the model. Especially in a training system for laparoscopic surgery, where the realism would determine the user's ability to transfer skills acquired in the VR system to an operating room. This therefore requires proper visualization of soft tissue and surgical tool interaction using mechanical and material properties of the soft tissue [13, 16, 17].

II. MATHEMATICAL MODEL FOR SOFT TISSUE MANIPULATION

VR training systems demonstrate a lot of promise in medical training but, also, have room for improvement, in order to increase the likelihood of skill transference from the training system to an operation room (OR). The mechanics of soft tissue play an important role in the development of soft tissue models in the VR for accurate representations of the interactions and manipulations of the soft tissue with the environment and the tools. There are two components of the soft tissue models that are affected by the mechanical properties of the soft tissue, which are the visual and haptic responses of the material. Therefore accurate representation of the mechanical and material properties of soft tissue in a training system, allows appropriate transference of the acquired skills into an OR environment. A previous study by the authors addresses the limitations of present soft tissue models for VR and the design of a new model based on the two dimensional wave equation [18, 19, 20]. The aim of the previous study was to increase the accuracy of the soft tissue model while limiting the computational cost.

A. Two-Dimensional Wave Equation

Engineers use two-dimensional wave equation, equation 1, to observe the structural integrity of beams, rods, cables, and plates when exposed to wave; for example the waves experienced during an earthquake. In these studies, the object of interest is homogeneous and isotropic [21, 22].

$$\frac{\delta^2 u}{\delta t^2} = c^2\left(\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2}\right); \quad c^2 = \frac{T}{\rho} \qquad (1)$$

In equation 1, a two- dimensional wave equation, T is the initial tension in the membrane, ρ is the density of the membrane, and the differential functions represent the solution's changes with respect to the inputs: x and y coordinates and the time variable [23].

In the previous study, the authors suggest the use of this mathematical model to mimic the behavior of the soft tissue during real-time manipulation in a VR system. This model is developed using mechanical and material properties of the soft tissue to appropriately represent the tissue of interest.

$$u_{mn}(x,y,t) = (B_{mn}\cos\lambda_{mn}t + B^*_{mn}\sin\lambda_{mn}t) \times \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (2)$$

Where $(m = 1, 2, 3\ldots)$,$(n = 1, 2, 3\ldots)$

$$B_{mn} = \frac{4}{ab}\int_0^b\int_0^a f(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dx\,dy;\ \text{where } \lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}\ \&\ c^2 = \frac{T}{\rho} \qquad (3)$$

The two dimensional wave equation solutions, equations 2-3, are used to develop the deformation based on the action of interest where, B_{mn} is the Euler formula and B^*_{mn} is the function that reflects the effects of the initial velocity [23]. The previous study uses the assumptions of the wave equation and relates them to the properties of the soft tissues. The developed model mimics the simple manipulation of pushing down on the soft tissue, resulting in a deformation, and the return of the soft tissue to its original shape upon the removal of the applied force [19, 20]. However, the wave equation can easily be modified to show other types of manipulations [20].

B. Design of a Heterogeneous Model

As discussed in the Introduction, skin is a heterogeneous material and consists of three main layers; epidermis, dermis, and subcutaneous fat. Heterogeneous materials, much like biological tissues, exhibit changing mechanical properties during displacement [24, 25]. Heterogeneous materials and models are studied extensively in mechanics of materials as these materials have superior material properties. The addition of multiple materials can enhance the original material by highlighting the desired mechanical properties from each component [25, 26].

Limited VR systems, presently, use heterogeneous models of soft tissues for real time manipulation, due to the computational cost associated with such models. Most heterogeneous models use FEA systems and simplify the mechanics of the soft tissue of interest [24, 26]. A study uses directional fields to determine the material distribution within a structure therefore creating a flow of the material (Figure 1) within the object [27]. Such models are not suitable for use in real-time simulations, like those of interest in this study.

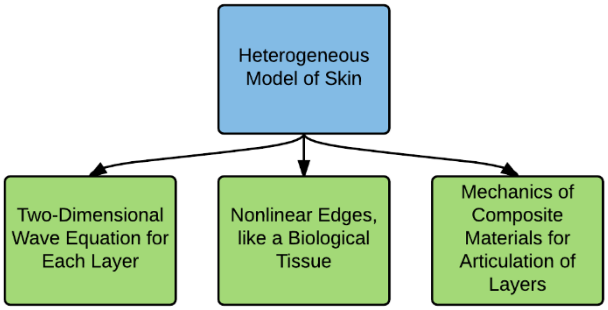


Figure 1. Components of the heterogeneous representation of skin

C. Nonlinear Tissue Edge

As heterogeneous materials change in mechanical and material properties during displacement through the material, the boundaries of each layer is important in determining which layers are affected during the deformation. Figure 2 shows a representation of the nonlinear boundaries that are used in this study, the figure is not drawn to scale, which is most similar to the boundaries seen between layers of skin in histological images that demonstrate these junctions. In this study, trigonometric functions are used to define the boundaries to represent the boundary between epidermis and dermis, and dermis and subcutaneous fat.

The equations and diagram shows a two-dimensional representation of the boundaries but they are applied to both axes (x-y) of the plane of the soft tissue, therefore forming a shape similar to that of an egg carton. These nonlinear boundaries are the first step in determining the affected tissue layers and therefore the constitutive mechanical behavior. Based on the depth of penetration by the tool through skin, and the functions for the boundaries, the affected layers are determined. Figure 3 lists out the possible combinations of affected layers.

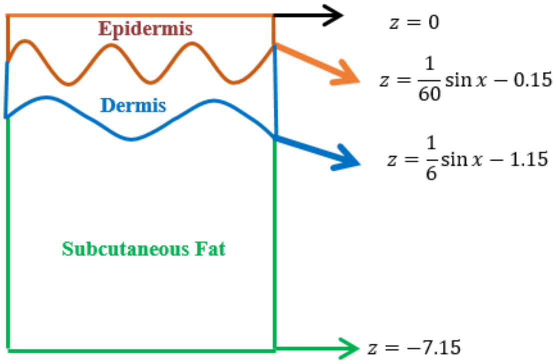


Figure 2. Graphical representation of the nonlinear edges between the three layers of skin

Figure 3. Flow of the possible layers affected during an interaction with a surgical tool in the order that they take place.

D. *Applying Composite Material Mechanics to Skin*

Composite materials are becoming very popular in various industries and have a lot of scope in the medical industry as representative of biological materials. Composite materials are composed of two or more components, which are put together to accentuate the best mechanical properties or behaviors of each component [28, 29].

This study uses composite material mechanics to define the behavior of multiple layers of the soft tissue during a deformation; therefore resulting in a weighted average relationship between the affected layers. Each layer of the skin is represented by the components of the composite material, and the mechanics is utilized to model the articulation between the different layers of the skin. The main assumption, all these calculations are based on, is that the stress at each layer is the same, as shown in equation 4. This assumption is made based on the Reuss model, which models each layer of the composite material as a blob, in which the material is manipulated perpendicular to the plane of the isotropic, heterogeneous material [30-33].

$$\sigma_{Epidermis} = \sigma_{Dermis} = \sigma_{Subcutaneous\ fat} \qquad (4)$$

Below, we demonstrate the method used to calculate the relationship between multiple layers and the exerted soft tissue force in response to manipulation by a laparoscopic tool.

1) *Epidermis-Dermis*

The first relationship observes the expected mechanical behavior of epidermis and dermis, when only the top two layers are manipulated. Stresses of the two layers are equal if skin is assumed to be a composite material and is based on the Reuss Model, which gives us the following relationships between the two layers.

$$\sigma_{Epidermis} = \sigma_{Dermis} \qquad (5)$$

Assuming the velocities,$\dot{x}_E = \dot{x}_D$ because the layers are moving together

$$k_E \Delta x_E A_D + b_E \dot{x} A_D = k_D \Delta x_D A_E + b_D \dot{x} A_E \qquad (6)$$

$$\Delta x_D = \frac{(b_D A_E - A_D b_E)\dot{x} + k_E \Delta x_E A_D}{k_D A_E} \qquad (7)$$

A is the area, k is spring constant, b is the damping constant This relationship is then simplified until the mechanical behavior is represented with respect to the displacement of the two tissues.

$$Depth = x_D + x_E = \frac{(b_D A_E - A_D b_E)\dot{x} + (k_E A_D + k_D A_E)\Delta x_E}{k_D A_E} \qquad (8)$$

Equation 8 combines the displacement of the two layers, which represent the total displacement of the tool in a real-time simulation, or the depth the tool has moved. A weighted average representation of the displacement is developed for the epidermis and dermis therefore the effect of the mechanical properties of the layers on the respective deformation. Equations 7 and 8, are used to determine the depth of deformation of the epidermis and dermis, which is then used to determine the shape of deformation using the two-dimensional wave equations that are discussed in [19, 20].

2) *Epidermis-Dermis-Subcutaneous Fat*

The steps discussed in the above section are used to determine the relationship between the epidermis, dermis and the subcutaneous fat. The functions representing the interaction are listed out in table 1.

TABLE I
Composite Model of Three Layers of Skin

Assuming $\dot{x}_E = \dot{x}_D = \dot{x}_S$	$Depth = \Delta x_E + \Delta x_D + \Delta x_S$ $\sigma_E = \sigma_D = \sigma_S$
Epidermis	Δx_E
Dermis	$\Delta x_D = \frac{(b_D A_E - A_D b_E)\dot{x} + k_E \Delta x_E A_D}{k_D A_E}$
Subcutaneous Fat	$\Delta x_S = \frac{(b_S A_E - A_S b_E)\dot{x} + k_E \Delta x_E A_S}{A_E k_s}$
$Depth = \Delta x_E + \frac{(b_D A_E - A_D b_E)\dot{x} + k_E \Delta x_E A_D}{k_D A_E} + \frac{(b_S A_E - A_S b_E)\dot{x} + k_E \Delta x_E A_S}{A_D k_s}$	

3) *Using Calculated Tissue Depths in the Two-Dimensional Wave Equation*

The relationships demonstrated in equations 5-10, are used to determine the depth of deformation of each layer based on their mechanical properties. This value is applied to the respective two-dimensional wave equation, for example, equation 8, which represents the deformation of the epidermis.

$$u_{mn}(x,y,t) = \left((6.11 \times 10^{-6})\cos(0.0032t)\right)\sin\frac{\pi x}{0.06}\sin\frac{\pi y}{0.06} \qquad (9)$$

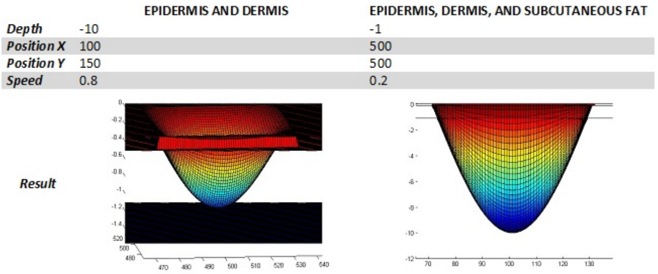
In this relationship, the depths are the output $u_{mn}(x,y,t)$ and the point of contact is the center of the deformation, which defines the x, y coordinate. Therefore, using variable t we can calculate the depths of the rest of the points along the size of the deformation(60×60).

4) Logic for Modeling Composite Material

1. Input of x & y (along the soft tissue plane) and z (along the vertical plane) position of the interactive tool and its speed are required.
   The position allows the system to determine whether an interaction has taken place or not.
   The soft tissue plane starts at z = 0 therefore any deformation will take place when the tool is touching the soft tissue at z < 0.
   The position of interaction along the plane of the soft tissue helps determine whether the manipulation is taking place along an edge, which would result in an incomplete deformation as the size of the deformation is 60x60.
2. Functions of the nonlinear boundary conditions are used to determine the affected layers of tissue and the extent of deformation at each layer.
3. One of the three categories from figure 4 is selected to determine the associated composite material mechanics.
4. Depth functions, as mentioned above, are used to determine the depth of deformation at each affected layer of tissue using the weighted averages.
5. The values from step 4 are used to calculate the deformation for each of the affected layers of soft tissue using the technique in subsection C.
6. The two-dimensional wave equation results for each layer are put together to form the resulting heterogeneous deformation.

E. Resulting Soft Tissue Model

Using the technique and relationships from subsection C, we develop deformations at any time variable, from the two dimensional wave equation solution, using the mechanics of each of the affected soft tissue layers. Figure 4 shows a cross sectional view of the resulting deformation in a heterogeneous material for deformations of the epidermis and dermis and epidermis, dermis and subcutaneous fat. Figure 5, also, shows how the depth of interaction can affect the layers that deform and therefore the mechanics that is utilized. The figure shows how the different layers come together to form the total deformation while being controlled by separate mechanical properties therefore different two-dimensional wave equations.



	EPIDERMIS AND DERMIS	EPIDERMIS, DERMIS, AND SUBCUTANEOUS FAT
Depth	-10	-1
Position X	100	500
Position Y	150	500
Speed	0.8	0.2

Figure 4. A cross sectional view of the three layers of skin during a deformation and their behavior. The layers are epidermis, dermis, and subcutaneous fat, from top to bottom, respectively

III. VALIDATION STUDY

The validation is based on the method designed in a previous study by S. Patel and J. Suthakorn [19, 20]. Various studies conclude that FEA is one of the most accurate methods of modeling soft tissue or a deformable material therefore the model is used as a baseline to evaluate the new heterogeneous representation of skin.

For this study the FEA setup from the previous study, [19, 20], is used and consists of the three layers of skin, similar to the wave equation representation. In this setup, the model is deformed by a force that rises over time until 5N after which the manipulation is removed until the model returns to its original shape. The same manipulation is modeled using the two dimensional wave equations. Firstly, the maximum deformation is found for the FEA model. Then, the value is inputted into the heterogeneous wave equation model to see whether the same deformation value is outputted by the function.

The computation time for the FEA model is 4 minutes whereas the heterogeneous wave equation was solved in 10 seconds. The results are analyzed using two statistics methods, which are normal distribution and two-sample t-test, which are used to determine whether the FEA and wave equation heterogeneous models are similar, which is expected as they both model the same soft tissue, the skin.

A. Two-Dimensional Wave Equation

Figure 5 shows the maximum deformations for the FEA model and the respective calculated maximum deformation for the heterogeneous wave equation model. As can be seen from the plot the deformations are very similar, but they are not equal with an average percent error of 0.089%.

B. Statistical Analysis

Figure 6 shows the normal distribution for the two models, FEA and wave equation. Based on the normal distribution results it is concluded that the two models are very similar with respect to their means and standard deviations. Likewise, table II, which shows the two-sample t-test results, supports the results from the normal distribution study.

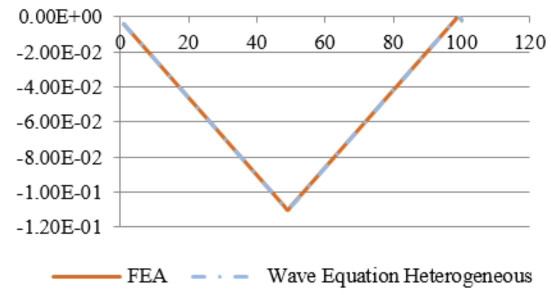


Figure 5. This plot shows a comparison between the maximum displacements in the two soft tissue models of interest.

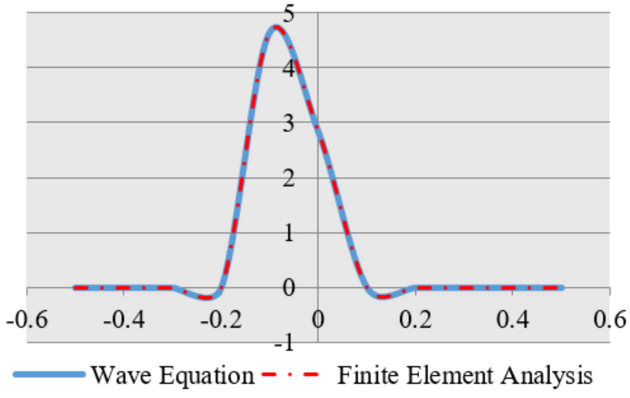


Figure 6. Normal distribution to compare the results from the heterogeneous model using wave equation vs. finite element analysis

TABLE II
Two-Sample T-Test Results

	FEA	Wave Equation
Mean	-0.05508	-0.05509
Variance	0.001022	0.001023
Observations	100	100
Pooled Variance	0.001022	
Hypothesized Mean Difference	0	
df	198	
t Stat	0.00355	
P(T<=t) one-tail	0.498586	
t Critical one-tail	1.652586	
P(T<=t) two-tail	0.997171	
t Critical two-tail	1.972017	

IV. DISCUSSION

A. Assessing the Overall Technique

This study aims to use a validated soft tissue model, the two-dimensional wave equation, to further improve the model of the soft tissue by developing a heterogeneous material. In the previous studies, the authors cover the importance of the mechanical and material properties of the different layers of skin that work together to develop the material property of the soft tissue. Therefore in this study, the authors introduce a new technique for modeling a heterogeneous representation of skin using multiple two dimensional wave equations, which represent the behavior of three layers of skin; epidermis, dermis, and subcutaneous fat. This is important, as present heterogeneous representation of soft tissue in a computer-based model is not suitable for real-time simulation due to the computational cost or accuracy, both of which are proportionally related.

As discussed in the previous study, the two-dimensional wave equations take into consideration the mechanical and material properties of each of the different layers of skin to accurately mimic their reaction to the manipulation of pushing on the soft tissue and then removing the manipulation for the return of the tissue's shape to its original shape.

The method used to articulate the different layers of two-dimensional wave equation, is the use of composite material mechanics to modify the mathematical models and incorporate the behavior of multiple layers of the material, in this case the skin. The mathematical model and mechanical relationship that is used to model the composite material considers the soft tissue is isotropic, which is a simplification as biological tissues are both heterogeneous and anisotropic. Therefore, through further studies into the directionality of fibers within each layer of the skin would be beneficial in further improving the accuracy of the soft tissue.

Though the model in this study is a heterogeneous representation of skin, it does not take into consideration the smaller components of skin, for example hair follicle or sweat glands. A future study into the effect of the location of such components on the mechanics of the soft tissue and its manipulation would help to further improve the heterogeneous model.

Just like the two-dimensional wave equation representation of each of the layers separately, the heterogeneous representation is not computationally expensive, because the deformation is calculated within seconds from the time an input is provided to the model. To reduce the computational cost, the accuracy is not compromised. This is because mechanical properties are used for the deformation function and a heterogeneous representation of the soft tissue is developed, both of which are considered to be hindering factors for soft tissue models that can be manipulated in real time. This therefore suggests the applicability of the soft tissue model in various medical simulations; for example surgical training or surgical planning.

B. Validation Study: Statistical Analysis

As can be seen from figure 7, the two datasets, therefore the computer-based models of skin have high variability. This therefore suggests that the two datasets are similar, which is expected as they model the same soft tissue and the same type of manipulation.

TABLE III
Analysis of T-Test Results

	$p - value > 0.05$	$t\ stat < t\ critical$
Wave Eq vs. FEA	True	True

The two-sample t-test results are analyzed in table 3, which further supports the conclusion from the normal distribution study. There are two relationships that are used to determine whether the null hypothesis of the t-test can be rejected, which are: $p - value > 0.05$ and $t\ stat < t\ critical$ both of which are true for the datasets in this study. Therefore it is concluded that the null hypothesis can not be rejected, which is expected as two models of the same soft tissue should have equal variances. The average percent error is less than 0.1% therefore showing that the datasets are very close to each other and the error may be a result of carried over approximations within the wave equation models for each of the layers.

C. *Future Application*

This model is designed for the training system at BART LAB, which presently uses a simple mass-spring system to model the soft tissue. With this soft tissue model the training system will have an accurate soft tissue model with a low computational cost, which allows for real-time manipulation of the soft tissue model. The training system would provide surgeons with a platform for practicing the required psychomotor skills without risking the lives of patients.

In this study, the heterogeneous model of skin requires an input of the surgical tool's position to develop the associated deformation. Therefore, it would be an easy incorporation of the new heterogeneous soft tissue model into a training system. The requirement for a position input also takes into consideration the method of interaction of the user with the virtual environment. In this case, the user will be able to manipulate the virtual soft tissue using a haptic device like Sensable's Phantom Omni, which not only allows the user to manipulate objects in the virtual environment but also provides a force feedback. Force feedback is important in a medical simulation, for example, for training in laparoscopic surgery, because force feedback and visual information are the only datasets the surgeon has access to for understanding the operating workspace. This is why the accuracy of the soft tissue's mechanics and visualization are very important for the effectiveness of such a system and the user's ability to transfer the information acquired from the virtual environment to an operating room.

V. CONCLUSION

In conclusion, this study integrates a validated soft tissue model for real-time manipulation, a two-dimensional wave equation model, to develop a heterogeneous representation of skin. It is important to consider the varying mechanical properties of the different layers in the skin, which attribute to the behavior of skin during a manipulation. The heterogeneous model is based on the mechanics of a composite material. The validation study of the heterogeneous model compares it to a FEA model of skin. The results demonstrate the strength of the new heterogeneous model for a real-time computer based soft tissue model.

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