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A Path Generation Algorithm for Biopsy Needle Insertion in A Robotic Breast Biopsy Navigation System

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***Abstract*—Breast biopsy is a procedure to get tissue sample to identify breast cancer. A radiologist performs breast biopsy by inserting the biopsy needle on the trajectory path from their opinion and experience under ultrasound-image guidance. An accuracy of needle insertion is necessary to avoid accidents; therefore, high skill and experience of radiologist are needed. Breast biopsy navigation system, with graphical user interfaces and a passive robotic needle holder, was developed to enhance an efficiency of this procedure. An algorithm of trajectory path is calculated and implemented in this system to get high performance. This paper presents the new idea of trajectory path's algorithm by two mathematic theories: Lagrange Multipliers and Vector in three-Dimensional. This path is calculated and generated in the condition of the shortest distance between suspected lesion and breast surface on a hemisphere shape. The trajectory path is displayed and guided the radiologist on graphical user interfaces with 2D and 3D guidance. When the position of lesion is changed, the trajectory path is also changed from the algorithm that can be assured about the trajectory path's algorithm and guidance system. Moreover, the passive robotic needle holder is applied to control the biopsy needle during perform breast biopsy with effective mechanism to increase the breast biopsy performance.**

I. INTRODUCTION

Breast cancer and other abnormal disease play an important health hazard for all human in the world, especially breast cancer in women. 47.8% of all female cancers [1] have breast cancer and cause high mortality rate. Women with no critical breast problems should be reassured as fast as possible and women who already had cancer should be diagnosed without delay [2]. The highest efficiency of diagnostic accuracy for breast disease are achieved by three approaches [3]. The first approach is clinical examination; the second approach is imaging with fine-needle aspiration cytology (FNAC). The last approach is core biopsy of important breast abnormalities. The imaging that applied to diagnose breast cancer consists of mammogram, ultrasound, 3D-ultrasound [4, 5], MRI [6, 7] and CT modality. In each modalities can give different benefit depends on an objective and application. Screening breast cancer with vertical and horizontal planes is the mammogram procedure to early diagnose abnormal lesion. Ultrasound image is a next modality that always used for breast biopsy guidance system because of real-time guidance and no radiation. Percutaneous needle biopsy is a standard practice for removing the suspected lesion to examination. The radiologist chooses the trajectory path form ultrasound image with high effectiveness then inserts the biopsy needle under ultrasound image guidance in real-time. This procedure is normally used because of a cost effective, safe, less invasive and minimal scarring. However, interventional procedure with core needle biopsy under ultrasound guidance requires high skill and experience of radiologist.

The needle trajectory path is a one factor that needs radiologist's skill to make a decision. The mistake of needle trajectory path can leads to accidents and many biopsy times. Therefore a precision of needle insertion is important and necessary.

Most trajectory guidance is displayed on ultrasound image in 2D image [8]. The trajectory path represented the needle movement using electromagnetic tracking system. The radiologist moves the probe with biopsy guidance kit until the trajectory path intersect the suspected lesion. Clear Guide One [9] is another method for ultrasound guide intervention using computer vision. The Clear Guide navigation head tracks needles and provides real-time information about needle path to display on Clear Guide CORE unit

Our breast biopsy navigation system, as shown on Fig. 1, was developed to increase ability and proficiency of breast biopsy procedure.

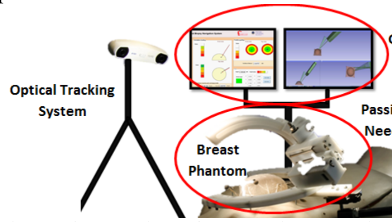

Fig. 1. The overview of Breast Biopsy Navigation System

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This system consists of 1) the graphical user interfaces on MATLAB and 3D Slicer, 2) the passive robotic needle holder, 3) breast phantom and 4) optical tracking system. The graphical user interfaces were developed in [10], and the passive robotic needle holder also developed and validated in [11]. However, trajectory path is needed to implement in this system for increasing high efficiency of breast biopsy navigation system.

Therefore, this paper presents a novel approach for the algorithm of trajectory path of breast biopsy in different idea with 3D guidance. The algorithm of the needle insertion path was applied using two mathematic theories. The first theory was “Lagrange Multipliers” [12]which was a mathematic optimization to find the maximum and minimum values of a function subject to equality constraints. The second theory was “Vector in a three-Dimensional surface” [13], which was used to find a linear vector space and another point that related to the linear vector.

The first section of this paper proposed the idea of breast feature which could represent in mathematical model and leaded to the algorithm of needle insertion path. The details of Lagrange Multipliers theory and vector in a 3-D space theory were explained. Then our algorithm base for hemisphere of breast, that was assumed to be a breast model completed from mammogram data, 3D-ultrasound or real-time CT, were applied to generate trajectory path in 3D space. Then breast phantom was designed and developed to test the experiment of this algorithm and breast biopsy navigation system. Graphic user interface (GUI on MATLAB and 3D Slicer), an important part for displaying the trajectory path and guiding the radiologist, was explained. Moreover, a passive robotic needle holder to control needle insertion on straight line was shown in next section. The last section described about discussion, conclusion and future work.

II. BREAST MODEL SELECTION

Breast feature should be considered before creating the algorithm of trajectory path. Method to validate breast feature for breast surgery and cosmetic was to measure the parameters of interest, namely, volume, surface area, and maximum vertical projection[14]. The measurement of breast volume and breast surface in prone position was measured by optical method. Breast model was computerized by comparing the digital image and photographs, and grid squares within the boundary. Another type for analyzing and synthesis of breast models was 3D scan data using a phase-shifting-moire topography scanner[15]. Their breast modeling needed principle component analysis (PCA) to find the direction of the largest variation among multiple shapes and parameters such as surface distance between bust point and bottom breast point, surface distance between bust point and inner breast point and etc. However, this template model had been designed with the landmark locations of surface measure remain the same location without deformation. [16]is a research that tried to represent breast model with a two half paraboloid facing each other and lying on the same elliptical base. A set of two digital mammogram views were used and developed to calculate the volume. By the way, in the experiments of screening breast cancer and breast biopsy usually made breast phantom with basic feature such as cube [17]and hemisphere shape [18].

III. TRAJECTORY PATH ALGORITHM

Trajectory path for breast biopsy needed 2 parameters to apply in the algorithm. The first was a position of suspected lesion and the second was breast surface. The position of suspected lesion and breast surface could be extracted from many types of medical image such as mammogram, 3D-ultrasound or real-time CT. For breast surface, the breast contour is similar to hemisphere shape, so the breast surface was represented by hemisphere equation. Therefore, the algorithm of trajectory path for breast biopsy guidance was calculated by assuming breast to be hemisphere shape with known position of the suspected lesion. The algorithm that used in this part was Lagrange Multipliers theory. Size of hemisphere shape was varying on the diameter of patient’s breast, and size of lesion could be adjustment depends on the stage of breast cancer. After localize the position of cancer, the trajectory path was calculated in the condition of the shortest distance between the breast surface and breast lesion. This trajectory path could decrease tissue damaged and trauma. The point on the breast surface that maked the shortest needle insertion path,was called an entry point of needle tip. For an orientation of biopsy needle,another point was computed from a vector that has the same direction as the trajectory path between the lesion and the entry point. The magnitude of this vector was equal to needle’s length.

A. *To calculate the position at the entry point*

The position at the entry point on breast surface was calculated using the Lagrange multiplier method. This is the method to find maximum and minimum of function which relate to another constrain function. The critical point of a multivariate function $(x; y; z)$ for 3 variables, subject to the constraint $g(x; y; z) = C$, was considered. f and g are functions with continuous first partial derivatives on the open set containing the curve $g(x; y; z) = C$. This theorem can be used in the condition of $\nabla g \neq 0$ at any point on the curve (∇ is the gradient operator). Then a problem could be solved as the following equation.

$$\nabla f(x; y; z) = \nabla\lambda g(x; y; z) \quad (1)$$
$$\text{And} \quad g(x; y; z) = C \quad (2)$$

Since ∇f and ∇g are vectors, three equations could be written as

$$f_x(x; y; z) = \lambda g_x(x; y; z) \quad (3)$$
$$f_y(x; y; z) = \lambda g_y(x; y; z) \quad (4)$$
$$f_z(x; y; z) = \lambda g_z(x; y; z) \quad (5)$$

Where $f_x = \frac{\partial f}{\partial x}$, $f_y = \frac{\partial f}{\partial y}$, $f_z = \frac{\partial f}{\partial z}$, $g_x = \frac{\partial g}{\partial x}$, $g_y = \frac{\partial g}{\partial y}$, $g_z = \frac{\partial g}{\partial z}$

λ is a dummy variable called "a Lagrange multiplier". When the value of this variable was found, 2 variables was presented. Then substitute both of them in (3), (4) and (5). The largest of these values is the maximum value of $f(x;y;z)$. On another hand, the smallest of these values is the minimum value of $f(x;y;z)$.

For trajectory path guidance, the minimum value of $f(x,y,z)$ was the minimum distance between the lesion and breast surface, so P(x, y, z) was found to make $f(x,y,z)$ to be the smallest value on the constraint surface $g(x,y,z)$.The constrain surface was represented by a sphere equation as shown on (6). Next equations were calculated with 4 assumptions:

1) Breast feature could be represented in a hemisphere shape
2) x_0, y_0 and z_0 were a hemisphere origin.
3) x_1, y_1 and z_1 were coordinates for the position of suspected lesion.
4) x, y and z arerepresented the position on the hemisphere surface and made the shortest distance between the hemisphere surface and suspected lesion. All of these conditions are shown on Fig. 2.

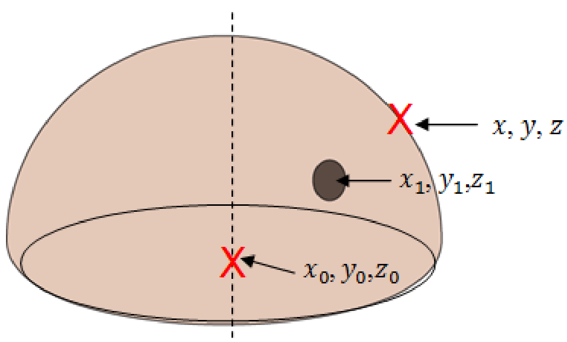


Fig. 2.The hemisphere origin (x_0, y_0 and z_0), the lesion's position (x_1, y_1 and z_1) and the position on the hemisphere surface that make the distance between the hemisphere surface and cancer to be the smallest

f rom equation (2), we could write an equation in term of hemisphere shape;

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2 \quad (6)$$
$$\nabla g(x,y,z)=2(x-x_0)i+2(y-y_0)j+2(z-z_0) \quad (7)$$

And

$$f(x,y,z)=(x-x_1)^2+(y-y_1)^2+(z-z_1)^2 \quad (8)$$
$$\nabla f(x,y,z)=2(x-x_1)i+2(y-y_1)j+2(z-z_1)k \quad (9)$$

Using the Lagrange multiplier theorem in (1),

$$2(x-x_1)=2\lambda(x-x_0) \quad (10)$$
$$2(y-y_1)=2\lambda(y-y_0) \quad (11)$$
$$2(z-z_1)=2\lambda(z-z_0) \quad (12)$$

From (7), (8) and (9), x, y and z are found to be

$$x=\frac{\lambda x_0-x_1}{\lambda-1} \quad (13)$$
$$y=\frac{\lambda y_0-y_1}{\lambda-1} \quad (14)$$
$$z=\frac{\lambda z_0-z_1}{\lambda-1} \quad (15)$$

Then substitute (13), (14) and (15) into (6) to find λ:

$$(\frac{\lambda x_0-x_1}{\lambda-1}-x_0)^2+(\frac{\lambda y_0-y_1}{\lambda-1}-y_0)^2+(\frac{\lambda z_0-z_1}{\lambda-1}-z_0)^2=r^2 \quad (16)$$
$$(\frac{x_0-x_1}{\lambda-1})^2+(\frac{y_0-y_1}{\lambda-1})^2+(\frac{z_0-z_1}{\lambda-1})^2=r^2 \quad (17)$$
$$\frac{(x_0-x_1)^2+(y_0-y_1)^2+(z_0-z_1)^2}{r^2}=(\lambda-1)^2 \quad (18)$$
$$\lambda_{1,2}=1\pm\frac{\sqrt{(x_0-x_1)^2+(y_0-y_1)^2+(z_0-z_1)^2}}{r} \quad (19)$$

After that, substituted $\lambda_{1,2}$ in equation (13), (14) and (15), then $(x_{\lambda1}, y_{\lambda1}, z_{\lambda1})$ and $(x_{\lambda2}, y_{\lambda2}, z_{\lambda2})$ were calculated. Finally, both of them were substituted into $f(x,y,z)$, and the position which made $f(x,y,z)$ to be the smallest position was the entry point. The distance between that point on breast surface or the entry point and suspected lesion could be calculated from $\sqrt{f(x,y,z)}$. However, if the breast shape is implied to be other type of geometry, the equation of $g(x,y,z)$ will be changed depends on that geometry.

This algorithm could be proved in this application by this experiment. The breast hemisphere with radius 55 mm. was assumed to calculate breast surface, then the breast lesion at X=20 mm, Y=30 mm and Z=40 mm was assumed to find the distance between breast lesion and breast surface. The algorithm calculated the shortest path between breast lesion and breast surface when the position of breast surface is X= 20.426, Y=30.64 and Z= 40.85302 which was represented in P1. The shortest path was 1.148 mm. After that, the distances between breast lesion and other positions of breast surface near the first position were calculated to check the result. The results of this experiment were shown on TABLE I and relationships between the distance and difference position on breast surface were shown on Fig.3.

TABLE I
THE DISTANCE BETWEEN DIFFERENT POSITIONS OF BREAST SURFACE AND BREAST LESION IN THE CONDITION OF BREAST HEMISPHERE WITH RADIUS 55MM. AND BREAST LESIONS AT X= 20MM Y= 30MM AND Z= 40MM

	Position on Breast Surface (mm)			Distance (mm)
	X	Y	Z	
P1	**20.426**	**30.64**	**40.85302**	**1.148352**
P2	20.425	30.65	40.84601	1.148418
P3	20.425	30.63	40.86101	1.148421
P4	20.427	30.63	40.86001	1.148414
P5	20.427	30.65	40.84501	1.148424
P6	20.424	30.62	40.86901	1.14863
P7	20.424	30.66	40.83901	1.148613
P8	20.428	30.62	40.86701	1.148603
P9	20.428	30.66	40.83701	1.148637

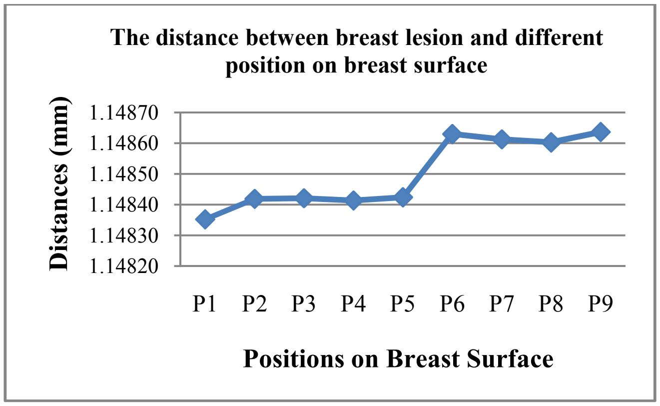


Fig. 3. Graph showed the relationship of distances between different positions on breast surface and breast lesion

In Fig. 3, P1 represented the position on breast surface that gave the shortest distance between breast lesion and breast surface. When the position on breast surface was changed the distance between them was also increasing. Therefore, the position on breast surface that was calculated from Lagrange Multiplier method could be applied in this application to find the shortest path between the surface of hemisphere breast and breast lesion.

B. To calculate the orientation at the entry point

Vector in 3D space or vector calculus plays an important role in differential geometry and partial differential equation. Euclidean distance in 3D space can be represented in vector. The vector is generated from the original point to the end point, which can explain a magnitude and direction. Therefore, vector in 3D space was applied in this method to find and explain the position and orientation of biopsy needle respect to trajectory path.

The point on breast surface from calculation or entry point was used to be the origin point of this vector. Then the point of this vector which led this vector parallel to the trajectory path and has the same magnitude with biopsy needle's length was found. Fig. 3 showed the position of lesion (P), entry point or the biopsy needle's tip (Q) and the end point of this vector or the end of the biopsy needle (R), while the magnitude of vector $\overrightarrow{QR}$ was equal to the biopsy needle's length. Therefore, the steps, to find this vector, were shown subsequently.

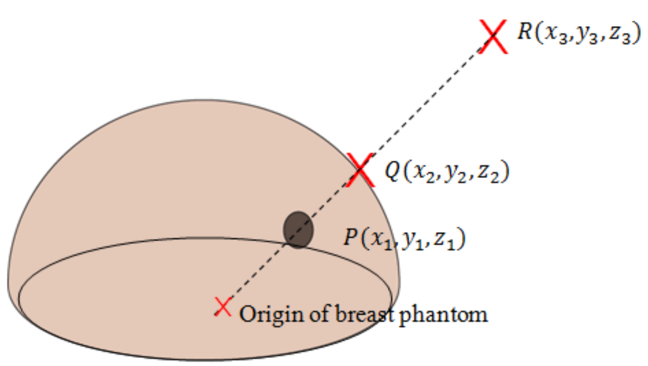


Fig. 4. The position of cancer (P), entry point (Q) and the end point of vector (R)

Let $P=(x_1,y_1,z_1)$, $Q=(x_2,y_2,z_2)$ and $R=(x_3,y_3,z_3)$. Therefore, Vector $\overrightarrow{PQ}$ is

$$\overrightarrow{PQ}=\begin{bmatrix}x_2-x_1\\ y_2-y_1\\ z_2-z_1\end{bmatrix} \quad (17)$$

And the magnitude of vector $\overrightarrow{PQ}$ is

$$\|\overrightarrow{PQ}\|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \quad (18)$$

The unit vector of vector $\overrightarrow{PQ}$was needed to be found for finding its direction. The unit vector of vector $\overrightarrow{PQ}$could be found from $\frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|}$.

Next, vector $\overrightarrow{QR}$was found with the same direction of vector $\overrightarrow{PQ}$ using vector's properties with normal vector. The magnitude of vector QR could be represented by $\|\overrightarrow{QR}\|$.

$$\frac{\overrightarrow{QR}}{\|\overrightarrow{QR}\|}=\frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} \quad (19)$$

$$\overrightarrow{QR}=\frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|}\times\|\overrightarrow{QR}\| \quad (20)$$

To find the end point of this vector, $R(x_3,y_3,z_3)$

$$\overrightarrow{QR}=\begin{bmatrix}x_3-x_2\\ y_3-y_2\\ z_3-z_2\end{bmatrix} \quad (21)$$

IV. Breast Phantom

Phantom is an object that is used instead of real-things during an experiment. There are many types of phantom depend on the application. The example is a gelatin phantom for learning sonographically guided interventional breast radiology [19]and silicone phantom for breast phantom [20]. The use of phantom helps developing Hand-Eye coordination and increasing performance. Moreover, it allows a radiologist or a student to do phantom practice instead of patient practice on their first training course.

This phantom was to simulate breast with breast lesion. However, the phantom was restricted with non-deformable property. The non-deformable breast means the lesion would not change its position during needle insertion. Therefore, the lesion's position had to be fixed. Phantom material with transparent and insertable was needed to be to investigate a process of needle insertion and checked an accuracy of our algorithm of trajectory path and breast biopsy navigation system. In addition, the radiologist can use the ultrasound to imaging this phantom if ultrasound system will implement in the breast biopsy guidance in the future. Therefore, the materials of breast and lesion had to be different in property of acoustic penetration.

After many experiments a lot of iterations of breast phantom were done and developed. Fig.5 showed the last iteration of our breast phantom. The breast phantom was made from gel candle because of reusable, transparent and insertable. Size of this breast phantom was 12 centimeters diameter, which was equal to a hemisphere mold.

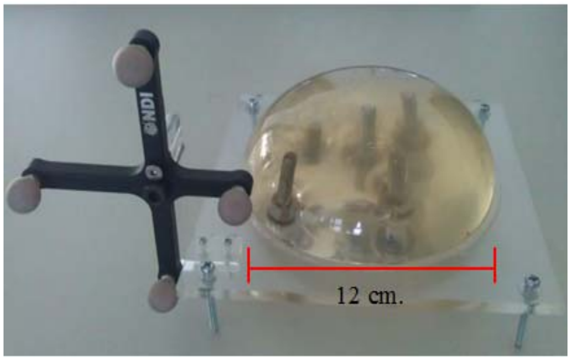


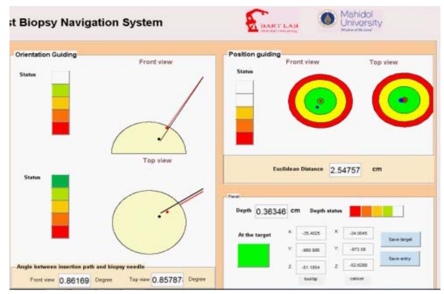
Fig. 5. Hemisphere breast phantom with many lesions in different positions

The experiments of breast biopsy had to perform many times and sometimes many days; therefore, gel candle was good solutions for breast phantom. Moreover, the sound wave could penetrate this material and display on ultrasound image. Lesions in breast were assumed with screw technique. The position of each lesion was fixed with known position. In the experiment, therefore, the users could perform breast biopsy many times without changing phantom. The most important to do this phantom was to avoid bubbles occurred.

V. The Algorithm on Graphic User Interface

Graphic user interfaces that consisted of GUI on MATLAB and 3D-Slicer were added with trajectory path guidance to navigate breast biopsy procedure. Thought ultrasound image has many advantages that mention above, but ultrasound is a two-dimensional technology functioning in three-dimensional environment. Therefore, 3D guidance plays an important role to increase radiologist's performance. 3D-Slicer is a free and open software package for medical image analysis. Medical images from various modalities such as MRI image, CT image and 3D-ultrsound image are added in this program. Moreover, 3D Slicer can display medical navigation in many applications such as neurosurgery guidance and spine surgical guidance. Our graphical user interfaces not only displayed the path generated but also had caution systems for guiding the radiologist. Fig. 6 showed our graphical user interfaces on GUI: guide in 2D image, and 3D Slicer: guide in 3D image.

In Fig. 6a, the interface showed two views: front view and top view and guided the radiologist to move the needle (red line) to the trajectory path (black line) with many light steps; a green light means acceptable. The guidance system guided the radiologist for both position and orientation of needle, trajectory path and breast phantom. Another part of GUI guidance on MATLAB was the relationship of lesion and needle's tip. The green sigh would appear after the distance between them was lower than our limitation or the biopsy needle on the lesion. On Fig. 6b, the graphical user interface guided the radiologist in a 3D image that was easier for the radiologist to understand. The radiologist could move the needle into a cone shape, which helped the radiologist to go to the generated-path faster and easier. When the position of lesions was changed, the trajectory path was also altered to find the shortest distance between breast surface and the lesion target. The view perspective could be adjusted depends on the radiologist or the users. The trajectory path was changed relate to the position of lesion, which can be proved from the graphical user interfaces and breast phantom. Fig. 7 showed the different of trajectory path on both graphical user interface on MATLAB and 3D Slicer when the cancer's position was changed. In the other hand, the trajectory path is still displayed in the same path as Fig.6 if the position of breast cancer is similar with cancer's position on that figure.



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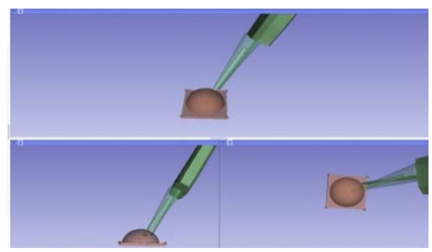

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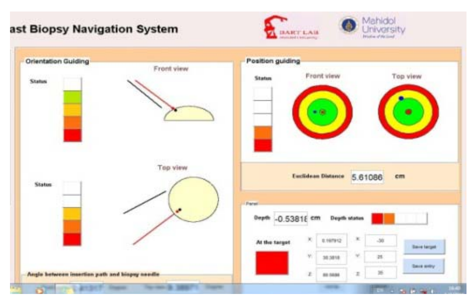
Fig. 5. Graphic User Interface on a) MATLAB b) 3D Slicer



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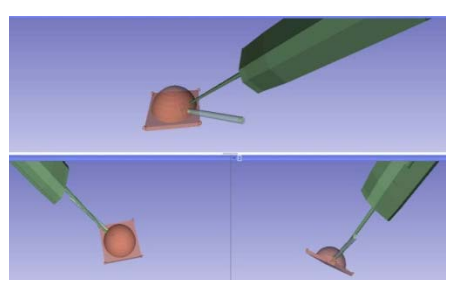

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Fig. 6. Graphic User Interface with different suspected lesion's positionon a) MATLAB b) 3D Slicer

VI. A Passive Robotic Needle Holder

Trajectory path was calculated with straight line from suspected lesion and breast surface, so the radiologist has to control needle insertion procedure in straight line with steady hand in free-hand method. The passive robotic needle holder as showing on Fig. 8was applied in this procedure.

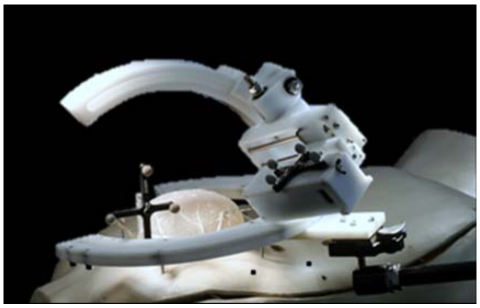

Fig. 8.The passive robotic needle holder with 2D linkage 3D holder

Feature of this robot imitated breast shape with hemisphere shape. The first and second joints were curvature translation, which made the robot moving around breast surface. The third and fourth joints had the same mechanism called revolute joint. Both of them helped needle's adjustment on trajectory path. The last joint controlled needle insertion in straight line with linear translation. Therefore, the radiologist, who performed breast biopsy with this robot, could control the needle insertion easier than free-hand method. The whole robot was made from Polyoxymethylene (POM) or Polyacetal to help friction movement; no electric sources were needed.

VII. Conclusion

Breast biopsy is the important procedure for early detection of breast cancer and gets the suspected lesion for examination. An accuracy of needle insertion can avoid unexpected accident and many biopsy times. An algorithm to generate trajectory path was presented with two mathematics theory; Lagrange Multipliers theory and vector in a 3-D space theory based on the shortest distance between suspected lesion and breast surface in hemisphere shape. The shortest needle insertion could decrease the rate of tissue damage and procedure time. This is the new idea for breast biopsy guidance system in 3D space with graphical user interfaces on 2D and 3D guidance. Moreover, high precision of needle insertion on trajectory path was increasing with the passive robotic needle holder, which helped the radiologist to control the biopsy path. All algorithms were approved with the breast phantom that related to graphical guidance system. However, [16]investigated many types of breast model and represented breast as a paraboloid structure having two half paraboloid placed on the same elliptical based. Therefore, the next trajectory path guidance should be calculated byparaboloid structure and implement the information of breast feature from mammogram image. The trajectory path from this method will has more realistic.

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